Cosmology with the D*p*-brane gas

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We study the effect of the Dp-brane gas in string cosmology. When one kind of Dp-brane gas dominates, we find that the cosmology is equivalent to that of Brans-Dicke theory with perfect-fluid-type matter. We obtain γ, the equation of state parameter, in terms of p and the space-time dimensions.

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Recent developments of string theory suggest that, in a Planck length regime, the quantum fluctuation is very large so that the string coupling becomes large and consequently the fundamental string degrees of freedom are not weakly coupled *good* ones [1]. Instead, solitonic degrees of freedom such as p-branes or Dp-branes [2] are more important. Therefore it is a very interesting question to ask what the effect of these new degrees of freedom on spacetime structure is, especially whether including these degrees of freedom resolves the initial singularity, which is a problem in standard general relativity.

What should be the starting point for investigation of the p-brane cosmology? It should be a generalization of general relativity. Brans-Dicke theory is a generic deformation of general relativity allowing variable gravity coupling. In fact the low energy theory of the fundamental string [3] contains Brans-Dicke (BD) theory with a fine-tuned deformation parameter ($\omega=-1$). Moreover, Duff and *et al.* [4] found that the natural metric that couples to the p-brane is the Einstein metric multiplied by a certain power of the dilaton field. In terms of this new metric, the action that gives the p-brane solution becomes the BD action with definite deformation parameter ω depending on p. In our previous papers [5,6], we have studied this kind of BD model and found exact solutions depending on two arbitrary parameters. Furthermore, according to the range of parameters involved, we also have classified all possible behaviors of the BD cosmology and found solutions resolving the initial singularity problem for some regions [7]. However, it remains an open questions as to how to determine the equation of state parameter γ. In this paper, we will determine γ from the relation between this BD theory and string theory with a Dp-brane.

We start by reviewing BD theory in which perfect-fluid-type matter is included. Let us assume that our universe is a D-dimensional homogeneous isotropic one. The general BD action is given by

$$S=\int d^Dx\sqrt{-g}e^{-\phi}[\mathcal{R}-\omega\nabla_\mu\phi\nabla^\mu\phi]+S_m, \tag{1}$$

where ϕ is the dilaton field and S_m is the matter part of the action. The equations of motion of BD theory become [8,9]

$$\begin{aligned}\mathcal{R}_{\mu\nu}-\frac{g_{\mu\nu}}{2}\mathcal{R}=\frac{e^{\phi}}{2}T_{\mu\nu}+\omega\left\{\nabla_\mu\phi\nabla_\nu\phi-\frac{g_{\mu\nu}}{2}(\nabla\phi)^2\right\}\\+\{-\nabla_\mu\nabla_\nu\phi+\nabla_\mu\phi\nabla_\nu\phi\\+g_{\mu\nu}\mathcal{D}^2\phi-g_{\mu\nu}(\nabla\phi)^2\},\\0=\mathcal{R}-2\omega\nabla^2\phi+\omega(\nabla\phi)^2.\end{aligned} \tag{2}$$

$\mathcal{R}$ is the curvature scalar and the cosmological metric is given by

$$ds_D^2=-\mathcal{N}dt^2+e^{2\alpha(t)}\delta_{ij}dx^idx^j\quad(i,j=1,2,\ldots,D-1), \tag{3}$$

where $e^{\alpha(t)}$ is the scale factor and $\mathcal{N}$ is a (constant) lapse function. Now, we assume that all variables are functions of time t only; then the curvature scalar [10] in D dimensions is given by

$$\mathcal{R}=g^{00}\mathcal{R}_{00}+g^{ij}\mathcal{R}_{ij}, \tag{4}$$

where

$$\begin{aligned}g^{00}\mathcal{R}_{00}&=\frac{D-1}{\mathcal{N}}[\ddot{\alpha}+\dot{\alpha}^2],\\g^{ij}\mathcal{R}_{ij}&=\frac{D-1}{\mathcal{N}}[\ddot{\alpha}+(D-1)\dot{\alpha}^2],\end{aligned} \tag{5}$$

whit $\dot{\alpha}=\nabla_t\alpha$. With the equation of state $p=\gamma\rho$, the energy-momentum tensor of perfect-fluid-type matter is given by

$$T_{\mu\nu}=pg_{\mu\nu}+(p+\rho)U_\mu U_\nu, \tag{6}$$

where U_μ is the fluid velocity. Under the hydrostatic equilibrium condition, the energy-momentum conservation is

$$\dot{\rho}+(D-1)(p+\rho)\dot{\alpha}=0. \tag{7}$$

Using $p=\gamma\rho$, we get the solution

$$\rho=\rho_0e^{-(D-1)(1+\gamma)\alpha}. \tag{8}$$

In our previous papers, the parameters γ and ω were considered as free parameters. In this paper, however, γ will be related to the dimension of the world volume of the p-brane.

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If we consider only the time dependence, the equations of motion (2) and the energy-momentum conservation (7) follow from the action

$$S=\int dte^{(D-1)\alpha-\phi}\left[\frac{1}{\sqrt{\mathcal{N}}}\{-(D-2)(D-1)\dot{\alpha}^2+2(D-1)\dot{\alpha}\dot{\phi}+\omega\dot{\phi}^2\}-\sqrt{\mathcal{N}}\rho_0 e^{-(D-1)(1+\gamma)\alpha+\phi}\right]. \tag{9}$$

Now, we introduce a new time variable τ by

$$dt=e^{(D-1)\alpha-\phi}d\tau. \tag{10}$$

Then the action can be written as

$$S=\int d\tau\left[\frac{1}{\sqrt{\mathcal{N}}}[-(D-2)(D-1)\alpha'^2+2(D-1)\alpha'\phi'+\omega\phi'^2]-\sqrt{\mathcal{N}}\rho_0 e^{(D-1)(1-\gamma)\alpha-\phi}\right], \tag{11}$$

where a prime means the derivative with respect to the new time variable τ and ρ_0 is considered as a positive real constant. Note that the variation over the constant lapse function gives a constraint equation. So far we have discussed BD theory.

Now we consider the string cosmology by the D-dimensional effective action with n-form field strength that is coming from the appropriate compactification of ten-dimensional low energy effective string theory. In the Einstein frame, the action reads

$$S=\int d^Dx\sqrt{-g^E}\left[R-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{e^{-\chi(n)\phi}}{2n!}H_n{}^2\right], \tag{12}$$

where χ is given by

$$\chi^2(n)=4-\frac{2(n-1)(\tilde{n}-1)}{n+\tilde{n}-2}, \tag{13}$$

with the dual dimension $\tilde{n}=D-n$. From this action, we can obtain an action in another frame by a conformal mapping. The p-brane frame is defined by the conformal mapping

$$g^p{}_{\mu\nu}=e^{\chi(n)\phi/(n-1)}g^E{}_{\mu\nu}, \tag{14}$$

where $g^E{}_{\mu\nu}$ ($g^p{}_{\mu\nu}$) means the metric in Einstein (p-brane) frame. In this p-brane frame [4], the action is written as

$$S=\int d^Dx\sqrt{-g^p}e^{-(D-2)a(n)\phi/2(n-1)}\left[R-\omega\nabla_\mu\phi\nabla^\mu\phi-\frac{1}{2n!}H_n^2\right], \tag{15}$$

where ω becomes [4]

$$\omega=-\frac{(D-1)(n-3)-(n-1)^2}{(D-2)(n-3)-(n-1)^2}. \tag{16}$$

In four dimensions, the BD parameter ω is given by $\omega=-\frac{4}{3}$ for the 0-brane ($p=0$) and $\omega=-\frac{3}{2}$ for the instanton ($p=-1$), etc. Hence, the BD parameter ω can be varied according to the p-branes included. However, it is not clear whether the matters come from NSp- or Dp-branes in the p-brane frame.

In the string frame, by an appropriate conformal mapping, the action is given by

$$S=\int d^Dx\sqrt{-g}\left[e^{-\phi}\{R+\nabla_\mu\phi\nabla^\mu\phi\}-\frac{e^{m\phi}}{2n!}H_{\mu_1\cdots\mu_n}H^{\mu_1\cdots\mu_n}\right]. \tag{17}$$

Notice that the BD parameter ω is fixed as -1. For $m=-1$, an n-form field strength comes from the compactification of Neveu-Schwarz–Neveu-Schwarz (NS-NS) three-form in ten-dimensional theory. $m=0$ for n-form coming from the Ramond-Ramond (R-R) sector. The dual form in NS-NS sector is defined by [4]

$$*H=e^{-\phi}H, \tag{18}$$

for the solitonic NSp-brane, and $m=1$ for this case.

Under the ansatz

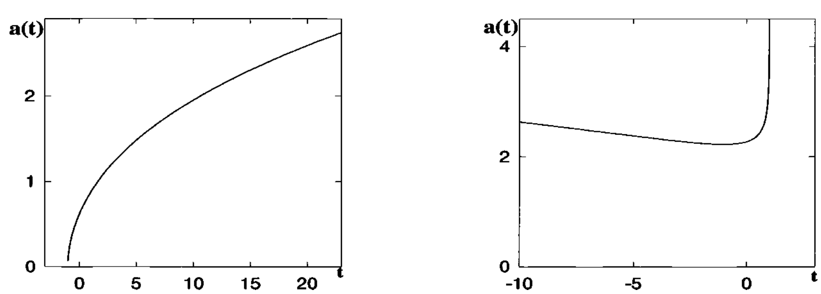

FIG. 1. The behavior of the scale factor with the D-particle.

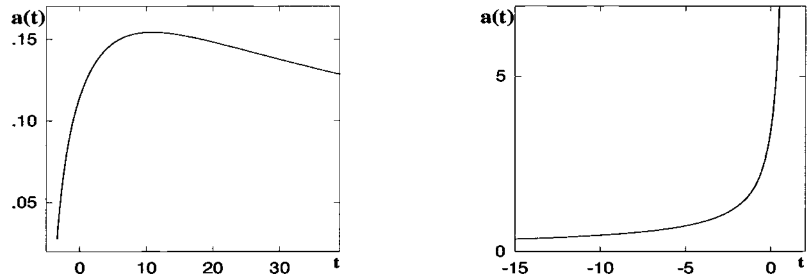

FIG. 2. The behavior of the scale factor with the D-string.

$$H_{\mu_1,\ldots,\mu_n}=\sqrt{\mathcal{N}}\sqrt{n}\epsilon_{\mu_1,\ldots,\mu_{n-1}}\nabla_0 A(t), \tag{19}$$

where $\mu_1,\ldots,\mu_{n-1}\neq 0$, the Bianchi identity

$$\nabla_{[\mu}H_{\mu_1,\ldots,\mu_n]}=0 \tag{20}$$

is always satisfied, since we assume that $A(t)$ is a function of t only. $\sqrt{\mathcal{N}}$ should be used for obtaining the correct constraint equation. In terms of $A(t)$, $\phi(t)$, and $\alpha(t)$, the action (17) for the cosmology becomes

$$S=\int dt e^{(D-1)\alpha-\phi}\left[\frac{1}{\sqrt{\mathcal{N}}}\{-(D-2)(D-1)\dot{\alpha}^2+2(D-1)\dot{\alpha}\dot{\phi}-\dot{\phi}^2\}+\frac{\sqrt{\mathcal{N}}}{2}\dot{A}^2 e^{-2(n-1)\alpha+(m+1)\phi}\right]. \tag{21}$$

Using the new time variable τ defined in Eq. (10), the action is given by

$$S=\int d\tau\left[\frac{1}{\sqrt{\mathcal{N}}}[-(D-2)(D-1)\alpha'^2+2(D-1)\alpha'\phi'-\phi'^2]+\frac{\sqrt{\mathcal{N}}}{2}A'^2 e^{-2(n-1)\alpha+(m+1)\phi}\right]. \tag{22}$$

From this, we can obtain the equations of motion

$$2(D-1)(D-2)\alpha''-2(D-1)\phi''-(n-1)\times(A')^2 e^{-2(n-1)\alpha+(m+1)\phi}=0, \tag{23}$$

$$2(D-1)\alpha''-2\phi''-\frac{(m+1)}{2}(A')^2 e^{-2(n-1)\alpha+(m+1)\phi}=0, \tag{24}$$

$$\nabla_\tau[A' e^{-2(n-1)\alpha+(m+1)\phi}]=0, \tag{25}$$

where the lapse function $\mathcal{N}$ is set to 1 after calculation. The solution of Eq. (25) is given by

$$A'=\sqrt{2}q e^{2(n-1)\alpha-(m+1)\phi}, \tag{26}$$

with a constant q. Substituting this into Eqs. (23) and (24), the resulting equations of motion are written as

$$2(D-1)(D-2)\alpha''-2(D-1)\phi''-2(n-1)q^2 e^{2(n-1)\alpha-(m+1)\phi}=0, \tag{27}$$

$$2(D-1)\alpha''-2\phi''-(m+1)q^2 e^{2(n-1)\alpha-(m+1)\phi}=0. \tag{28}$$

These equations of motion can be derived from the following action:

$$S=\int d\tau\left[\frac{1}{\sqrt{\mathcal{N}}}[-(D-2)(D-1)\alpha'^2+2(D-1)\alpha'\phi'-\phi'^2]-\sqrt{\mathcal{N}}q^2 e^{2(n-1)\alpha-(m+1)\phi}\right]. \tag{29}$$

Comparing this action with that of BD theory given by Eq. (11), the action of BD theory is equivalent to that of string theory with the Dp-brane gas $(m=0)$ if we set $\rho_0=q^2$ and $2(n-1)=(D-1)(1-\gamma)$. From these, the BD parameter γ is related to p, the spatial dimension of the world volume of the p-brane:

$$\gamma=\frac{D-2p-3}{D-1}. \tag{30}$$

This is the main result of our paper. Note that in string frame, the behaviors of the cosmology depend only on p $(=n-2)$. In four-dimensional space-time, γ of the instanton gas $(p=-1)$ is fixed to 1 and for the particle $(p=0)$ $\gamma=1/3$ is consistent with the known value of γ for the radiation dominant era. In our previous work [5], the behavior of

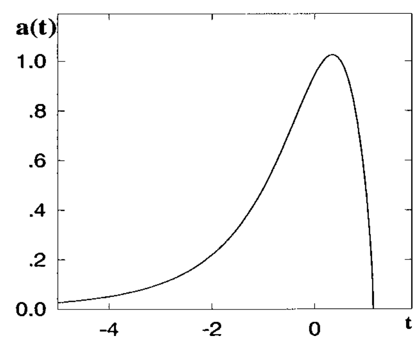

FIG. 3. The behavior of the scale factor with the Dp-brane ($p=2$ or $p=3$).

the scale factor for general γ and ω was classified. With the D-particle gas ($p=0$ and $\gamma=1/3$), the string cosmology has two phases as in Fig. 1. For the D-string gas, $\gamma=-1/3$ which is consistent with the value of the cosmic string gas [11], the behavior of the scale factor has two phases as shown in Fig. 2. For D2-brane or D3-brane cases ($p=2$ or $p=3$), the scale factor behaves as in Fig. 3.

In this paper, we have shown that string theory with the Dp-brane gas can be described by BD theory with perfect-fluid-type matter and the parameter γ in BD theory is determined by the dimension of the brane. In string theory with the NS- ($m=-1$) or the dual NS-brane ($m=1$) gas, the matter couples to the dilaton, and the energy-momentum of the perfect fluid is not conserved due to this coupling. Hence we cannot consider the NS-type brane gas as a perfect fluid. To describe the NS brane gas as a perfect fluid, we have to study string theory in a frame where the dilaton does not couple with the brane. Details will be discussed in a later work [12].

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[1] E. Witten, Nucl. Phys. **B460**, 335 (1996).
[2] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); J. Polchinski, S. Chaudhuri, and C. V. Johnson, hep-th/9602052; E. Witten, Nucl. Phys. **B460**, 335 (1996).
[3] G. Veneziano, hep-th/9510027; for string cosmology there are a vast number of references. Here we list some of the relevant ones and for more references: see Mod. Phys. Lett. A **8**, 3701 (1993), and references therein; G. Veneziano, Phys. Lett. B **265**, 287 (1991); M. Gasperini, J. Maharana, and G. Veneziano, Nucl. Phys. **B472**, 349 (1996); S.-J. Rey, Phys. Rev. Lett. **77**, 1929 (1996); Nucl. Phys. B (Proc. Suppl.) **52A**, 334 (1997); hep-th/9609115; M. Gasperini and G. Veneziano, Phys. Lett. B **387**, 715 (1996); R. Brustein and G. Veneziano, *ibid.* **329**, 429 (1994); E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D **50**, 4868 (1994); H. Lu, S. Mukherji, and C. N. Pope, *ibid.* **55**, 7926 (1997); A. Lukas, B. A. Ovrut, and D. A. Waldram, Phys. Lett. B **393**, 65 (1997); Nucl. Phys. **B495**, 365 (1997); **B509**, 169 (1998); S. Mukherji, Mod. Phys. Lett. A **12**, 639 (1997).
[4] M. J. Duff, R. R. Khuri, and J. X. Lu, Phys. Rep. **259**, 213 (1995).
[5] Chanyong Park and Sang-Jin Sin, Phys. Rev. D **57**, 4620 (1998); J. Korean Phys. Soc. **34**, 463 (1999).
[6] Sung-geun Lee and Sang-Jin Sin, J. Korean Phys. Soc. **32**, 102 (1998).
[7] S. K. Rama, Phys. Lett. B **408**, 91 (1997). For earlier graviton-dilaton models, see Phys. Rev. Lett. **78**, 1620 (1997); Phys. Rev. D **56**, 6230 (1997).
[8] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[9] G. Veneziano, Phys. Lett. B **265**, 287 (1991).
[10] H. Lü, S. Mukherji, and C. Pope, Int. J. Mod. Phys. A **14**, 4121 (1999).
[11] R. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1988); A. A. Tseytlin and C. Vafa, *ibid.* **B372**, 443 (1992); A. A. Tseytlin, Class. Quantum Grav. **9**, 979 (1992); N. Sanchez and G. Veneziano, Nucl. Phys. **B333**, 253 (1990); G. Veneziano, Phys. Lett. B **265**, 287 (1991); M. Gasperini, N. Sanchez, and G. Veneziano, Nucl. Phys. **B364**, 365 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1992); Mod. Phys. Lett. A **8**, 3701 (1993), and references therein.
[12] Sunggeun Lee, Chanyong Park, and Sang-Jin Sin (in preparation).